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Dirichlet Boundary State in Linear Dilaton Background

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Dirichlet-branes have emerged as important objects in studying nonperturbative string theory. It is important to generalize these objects to more general backgrounds other than the usual flat background. The simplest case is the linear dilaton condensate. The usual Dirichlet boundary condition violates conformal invariance in such a background. We show that by switching on a certain boundary interaction, conformal invariance is restored. An immediate application of this result is to two dimensional string theory.

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Polchinski's D-branes [1] represent an exact treatment of various p-brane solutions. Such objects become increasingly important in understanding nonperturbative aspects of string theory, in particular duality. The construction was immediately generalized to include bound states [2] [5], and boundary states associated to them are studied in [3] and [4]. The stringy properties of D-branes are explored in [6]. All these are perfectly in harmony with duality conjectures.

Properties of D-branes in a curved background have not been examined in much detail. The simplest nontrivial background is the linear dilaton condensate, which occurs in many interesting situations such as two dimensional string theory, and two dimensional black hole and related models. As being already pointed out in [7], the simple Dirichlet boundary condition does not respect conformal invariance in presence of a background charge. A mechanism proposed there is to use degenerate loops to compensate such violation. We shall adopt another strategy in this note to restore conformal invariance: Adding a certain boundary term. This may be connected to suggestion of [7], although we have not tried to explore this possibility. The necessity of such a boundary term indicates that in order to include D-branes in a linear dilaton background, certain open string background must be switched on. Perturbative closed strings do not sense the existence of such background, since diagrams involved are Riemann surfaces without boundaries. Some applications of our result to two dimensional string are being worked out in [8].

For simplicity, we consider only one free scalar on the world-sheet denoted by ϕ. The holomorphic component of the stress tensor reads

$$T=-\frac{1}{2}(\partial\phi)^2+Q\partial^2\phi, \tag{1}$$

and a similar anti-holomorphic counterpart. The central charge of this free scalar is $c=1+12Q^2$, and $Q=\sqrt{2}$ in two dimensional string theory. Consider a unit disk, the conformal invariance condition on the boundary is $Tdz^2=\bar{T}d\bar{z}^2$. In other words, there is no net energy-momentum flow out of the boundary in world-sheet point of view. It is convenient to work with mode expansions

$$\begin{aligned}\phi&=\varphi_0-ip(\ln z+\ln\bar{z})-i\sum_{n\neq 0}\frac{1}{n}\left(\alpha_{-n}z^n+\tilde{\alpha}_{-n}\bar{z}^n\right),\\ L_n&=[p+iQ(n+1)]\alpha_n+\frac{1}{2}\sum_{m\neq 0}\alpha_{m+n}\alpha_{-m},\end{aligned} \tag{2}$$

a similar formula for $\tilde{L}_n$. The commutators are $[\alpha_m,\alpha_n]=m\delta_{m+n,0}$, and similarly for the right-moving modes. Let $K_n=L_n-\tilde{L}_{-n}$. The boundary condition is entirely encoded in the boundary state $|B\rangle$, and the conformal invariance condition is $K_n|B\rangle=0$.

The usual Neumann boundary condition is given by $\partial_r\phi = 0$ on the boundary of the unit disk. In terms of the boundary state, it states that

$$p|B\rangle_N = (\alpha_n + \tilde{\alpha}_{-n})|B\rangle_N = 0.$$

Due to the existence of the background charge Q, one has to modify the boundary condition a bit: $p = -iQ$. So there must be a net momentum flow out of the boundary (in view of spacetime ϕ). One way to see this is to consider the commutators

$$[K_m, \alpha_n + \tilde{\alpha}_{-n}] = 2m(p + iQ)\delta_{m+n,0} - m\left(\alpha_{m+n} + \tilde{\alpha}_{-m-n}\right). \tag{3}$$

So when $p = -iQ$, the center term disappears, and it is possible to impose both the conformal invariance condition and Neumann boundary condition. Dirichlet boundary condition is $\partial_\theta\phi = 0$ on the boundary. As Polchinski already observed [7], this simple condition is no longer conformally invariant if $Q \neq 0$. Or under a general conformal transformation, $\delta\phi$ acquires a term proportional to the Weyl factor generally nonvanishing on the boundary. Another way to see this is to derive similar commutators as (3). In terms of modes, Dirichlet boundary condition is

$$(\alpha_n - \tilde{\alpha}_{-n})|B\rangle_D = 0.$$

This is not compatible with the conformal invariance condition since

$$[K_m, \alpha_n - \tilde{\alpha}_{-n}] = 2im^2Q\delta_{m+n,0} - m\left(\alpha_{m+n} - \tilde{\alpha}_{-m-n}\right). \tag{4}$$

The center term, independent of p, is always nonvanishing.

To restore conformal invariance, we have to modify the boundary condition. To be as close to the ordinary Dirichlet condition as possible, one requires that a net momentum transfer is possible if one scatters string states against the object described by the boundary state. So $|B, p\rangle$ is an eigen-state of p with arbitrary number p. To solve equations $K_n|B, p\rangle = 0$, it is convenient to adopt the coherent state technique introduced in [9]. Introduce the following coherent states

$$(\alpha_n - \tilde{\alpha}_{-n} - x_n)|x, p\rangle = 0, \tag{5}$$

where n can be either positive or negative. The Hermiticity condition $x_{-n} = \bar{x}_n$ must be met. This set of states forms a complete orthogonal basis. The solution to (5) is

$$|x, p\rangle = \exp\left(\sum_{n=1}^{\infty}\frac{1}{n}[-\frac{1}{2}x_n x_{-n} + \alpha_{-n}\tilde{\alpha}_{-n} + x_n\alpha_{-n} - x_{-n}\tilde{\alpha}_{-n}]\right)|p\rangle. \tag{6}$$

It can be checked that these states normalize to delta function.

We postulate that the desired modified Dirichlet boundary state is of the form

$$|B,p\rangle = \int [dx]|x,p\rangle \Phi(x). \tag{7}$$

To solve $K_n|B,p\rangle = 0$, one first computes

$$K_n|x,p\rangle = (2iQn\tilde{\alpha}_{-n} + \sum_{m>0}[x_{m+n}\alpha_{-m} + \bar{x}_{m-n}\tilde{\alpha}_{-m}] + (p+iQ(n+1))x_n + \frac{1}{2}\sum_{0<m<n} x_{n-m}x_m)|x,p\rangle$$

for $n>0$. A similar formula can be derived for $n<0$. Observing the form (6), one replaces α_{-m} in the above formula by $m\partial_m + \frac{1}{2}x_{-m}$ and $\tilde{\alpha}_{-m}$ by $-m\partial_{-m} - \frac{1}{2}x_m$. Thus K_n is replaced by a first order differential operator when acts on $|x,p\rangle$. Substituting this relation into (7) and integrating by parts, the conformal invariance condition for Φ is obtained

$$\left(2iQn^2\partial_n + (p+iQ)x_{-n} - \sum_{m=-\infty}^{\infty} mx_{m-n}\partial_m\right)\Phi(x) = 0, \tag{8}$$

incidentally this is valid for both $n>0$ and $n<0$. Assume $\Phi = e^K$, the differential equations for K follow from (8)

$$2iQn^2\partial_n K + (p+iQ)x_{-n} = \sum_{m=-\infty}^{\infty} mx_{m-n}\partial_m K. \tag{9}$$

We now solve differential equations (9). Observe that these equations can be solved recursively. Let $K = \sum_N K_N$, where K_N contains N-th powers in the x's. The constant term is not much of interest at present. There can be no linear term. The first term is K_2 satisfying

$$2iQn^2\partial_n K_2 + (p+iQ)x_{-n} = 0, \tag{10}$$

with solution

$$K_2 = -(p+iQ)(\frac{i}{2Q})\sum_{m,n}\frac{1}{2mn}x_m x_n \delta_{m+n,0}. \tag{11}$$

The recursive relation is then

$$2iQn^2\partial_n K_{N+1} = \sum mx_{m-n}\partial_m K_N. \tag{12}$$

The ansatz

$$K_N = a_N \sum_{m_i}\frac{1}{m_1\ldots m_N}x_{m_1}\ldots x_{m_N}\delta_{m_1+\ldots+m_N,0}$$

leads to

$$a_{N+1} = \frac{i}{2Q}\frac{a_N}{N+1} = -(p+iQ)(\frac{i}{2Q})^N\frac{1}{(N+1)!}.$$

Remarkably, the sum $\sum_N K_N$ is given by a very simple form, up to a constant term

$$\begin{aligned} K &= 2iQ(p+iQ)\oint\frac{d\theta}{2\pi}e^{-\frac{1}{2Q}X(\theta)}, \\ X(\theta) &= -i\sum_m\frac{1}{m}x_m e^{im\theta}. \end{aligned} \tag{13}$$

$X(\theta)$ is real-valued. When only a pair x_n and x_{-n} are nonvanishing, $X(\theta)$ is a sine function in θ and the phase of x_n.

Finally, the boundary state is given by

$$|B,p\rangle = \int[dx]|x,p\rangle\exp\left(2iQ(p+iQ)\oint\frac{d\theta}{2\pi}e^{-\frac{1}{2Q}X(\theta)}\right). \tag{14}$$

The following remarks on (14) are in order. The solution to the infinite set of differential equations (9) is by no means unique. However, we trust that the solution given by (14) is the appropriate generalization of the usual Dirichlet boundary state to the background of linear dilaton condensate, because not only the solution looks very elegant, but also it appears to return to the usual Dirichlet boundary state in the limit $Q\to 0$. In this limit, whenever $X(\theta)\neq 0$, the exponent $\int d\theta\exp(-X/(2Q)$ is large, so the integral in (14) tends to center at $X=0$ which is the usual Dirichlet state. As a consistency check, take $p=-iQ$, then $\Phi=1$. Integrating over x we obtain the Neumann boundary state discussed before. For a real Q, this is unphysical if we are interested in real momentum transfer.

As we have expected, the form of (14) tells us that a boundary operator which is to replace the "wave function" $\Phi(x)$ is needed in order to restore conformal invariance. What is a little surprising is that the coefficient $2iQ(p+iQ)$ is fixed for a given Q and p. If one attempts to replace x_m in $X(\theta)$ by $\alpha_m-\tilde{\alpha}_{-m}$, one obtains operator ϕ without the zero mode part. Again, integrating over the x's results in the Neumann boundary state, except that the zero mode part is that of Dirichlet. We conclude that the generalized Dirichlet boundary state in a linear dilaton background is obtained by applying a boundary operator

$$\exp\left(2iQ(p+iQ)\oint\frac{d\theta}{2\pi}e^{\frac{1}{2Q}\phi_{oc}}\right)$$

to the Neumann boundary state carrying momentum p. We note in passing that a similar interaction boundary term is studied in [10], where no background charge is introduced. The usual Dirichlet boundary state is achieved by letting the coupling constant of the

boundary interaction go to infinity. While such a limit is achieved by taking $Q \to 0$ in our case. One more interesting aspect deserves mentioning. For a real Q, the Liouville field ϕ can be formally viewed as having an imaginary radius $R = 2iQ$. It is just this pure imaginary radius appearing in $\Phi(x)$. Our derivation presented in this note is brute force in nature. The result looks quite elegant, so it appears that there is a direct derivation based on the usual conformal technique.

That the oscillator part of the boundary state is Neumann, despite the appearance of a boundary interaction which helps to suppress spread of oscillators, indicates that the object is not point-like in the transverse direction. This should have some interesting physical implication. Linear dilaton is a generic feature in conical phase transitions, and D-branes in such a background pose many interesting questions, for example, a new length scale [11]. Whether our boundary state will shed light on the question of a new scale in string theory remains to see.

Some applications of our main result (14) will appear in [8], here we only make a few preliminary comments. The boundary state with location at $\phi = \phi_0$ is obtained by Fourier transform

$$|B, \phi_0\rangle = \int dp e^{-ip\phi_0} |B, p\rangle. \tag{15}$$

Since the boundary wave function Φ involves p, the integration over p yields a delta function containing $\int d\theta \exp(-X/(2Q))$, showing that the object is not point-like in ϕ. The disk amplitude, important in evaluating nonperturbative effects, is given by

$$\langle B, \phi_0 | 0 \rangle, \tag{16}$$

or

$$\langle B, \phi_0 | \exp(-\mu \int d^2 z e^{-\sqrt{2}\phi}) | 0 \rangle, \tag{17}$$

when a tachyon condensate is present. The second object is not very easy to compute, and eventually one has to invoke the analytic extension method of [12]. Generalization to some supersymmetric situation is also under consideration, and there seems to be no intrinsic difficulty in doing this.

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